Table 5-1. Properties of the Fourier Transform

Property	Signal	Fourier transform
	$x(t)$	$X(\omega)$
	$x_1(t)$	$X_1(\omega)$
	$x_2(t)$	$X_2(\omega)$
Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(\omega) + a_2X_2(\omega)$
Time shifting	$x(t - t_0)$	$e^{-j\omega t_0}X(\omega)$
Frequency shifting	$e^{j\omega_0 t}x(t)$	$X(\omega - \omega_0)$
Time scaling	$x(at)$	$\frac{1}{\|a\|}X\left(\frac{\omega}{a}\right)$
Time reversal	$x(-t)$	$X(-\omega)$
Duality	$X(t)$	$2\pi x(-\omega)$
Time differentiation	$\frac{dx(t)}{dt}$	$j\omega X(\omega)$
Frequency differentiation	$(-jt)x(t)$	$\frac{dX(\omega)}{d\omega}$
Integration	$\int_{-\infty}^{t} x(\tau)\,d\tau$	$\pi X(0)\delta(\omega) + \frac{1}{j\omega}X(\omega)$

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Convolution	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
Multiplication	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(\omega) * X_2(\omega)$
Real signal	$x(t) = x_e(t) + x_o(t)$	$X(\omega) = A(\omega) + jB(\omega)$
		$X(-\omega) = X^*(\omega)$
Even component	$x_e(t)$	$\mathrm{Re}\{X(\omega)\} = A(\omega)$
Odd component	$x_o(t)$	$j\,\mathrm{Im}\{X(\omega)\} = jB(\omega)$

Parseval's relations

$$\int_{-\infty}^{\infty} x_1(\lambda)X_2(\lambda)\,d\lambda = \int_{-\infty}^{\infty} X_1(\lambda)x_2(\lambda)\,d\lambda$$

$$\int_{-\infty}^{\infty} x_1(t)x_2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_1(\omega)X_2(-\omega)\,d\omega$$

$$\int_{-\infty}^{\infty} |x(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\,d\omega$$

Table 5-2. Common Fourier Transforms Pairs

$x(t)$	$X(\omega)$
$\delta(t)$	1
$\delta(t-t_0)$	$e^{-j\omega t_0}$
1	$2\pi\delta(\omega)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$
$\sin\omega_0 t$	$-j\pi[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$
$u(t)$	$\pi\delta(\omega)+\dfrac{1}{j\omega}$
$u(-t)$	$\pi\delta(\omega)-\dfrac{1}{j\omega}$
$e^{-at}u(t), a>0$	$\dfrac{1}{j\omega+a}$
$te^{-at}u(t), a>0$	$\dfrac{1}{(j\omega+a)^2}$
$e^{-a\lvert t\rvert}, a>0$	$\dfrac{2a}{a^2+\omega^2}$
$\dfrac{1}{a^2+t^2}$	$e^{-a\lvert\omega\rvert}$
$e^{-at^2}, a>0$	$\sqrt{\dfrac{\pi}{a}}\,e^{-\omega^2/4a}$
$p_a(t)=\begin{cases}1 & \lvert t\rvert<a\\ 0 & \lvert t\rvert>a\end{cases}$	$2a\dfrac{\sin\omega a}{\omega a}$
$\dfrac{\sin at}{\pi t}$	$p_a(\omega)=\begin{cases}1 & \lvert\omega\rvert<a\\ 0 & \lvert\omega\rvert>a\end{cases}$
$\operatorname{sgn} t$	$\dfrac{2}{j\omega}$
$\sum_{k=-\infty}^{\infty}\delta(t-kT)$	$\omega_0\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega_0), \omega_0=\dfrac{2\pi}{T}$